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MHD Flow and Heat Transfer of A Jeffrey Fluid Over A Stretching Sheet With Viscous Dissipation

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ABSTRACT

The steady two-dimensional magnetohydrodynamic (MHD) boundary layer flow and heat transfer of a Jeffrey fluid over a stretched sheet in the presence of viscous dissipation is studied. The horizontal sheet is considered to have a non-isothermal temperature. The governing equations that govern the fluid flow and heat transfer are in the form of partial differential equations, which are then reduced to a set of non-linear ordinary differential equations by a similarity transformation. The resulting differential equations are solved numerically using an implicit finite difference scheme. The effects of Deborah number β, Eckert number Ec, magnetic parameter M and Prandtl number Pr on the flow and heat transfer characteristics are investigated.

Keywords: MHD, viscous dissipation, Eckert number, Jeffrey fluid

1. Introduction

The simplest rheological model of a fluid is a Newtonian fluid, which is governed by the Navier-Stokes equation. In many fields, such as bioengineering, the food industry and drilling operations, the fluids with long chains of molecules and which contain fine particles, exhibit non-Newtonian characteristics. As such, the traditional Newtonian fluid is inadequate to describe the dynamic flow of non-Newtonian fluids. There are few constitutive equations for the non-Newtonian fluids available in the literature and in return, the differential systems for such fluids are more complicated and much non-linear than those of the viscous fluid. Owing to its application in the industry and in the technological world, the non-linear fluid has attracted the attention of researchers in the last few decades. One of the non-Newtonian fluids is the Jeffrey model, which is a relatively simpler linear model using time derivatives instead of convected derivatives, which are used by most fluid models. There is little literature available for these types of fluids; some of them can be found in (Hayat and Mustafa, 2010), (Hayat and Obaidat, 2012), (Nadeem and Fang, 2011), (Turkyilmazoglu and Pop, 2013) and (Qasim, 2013).

The pioneer work of Sakiadis (1961), who performed the study for the flow induced by a moving plate, and Crane (1970), who examined the flow generated by a linearly stretching sheet, has opened doors for researchers to further investigate the effects of stretching on the boundary layer flow, involving different types of fluids and various aspects of boundaries and flow conditions. The study of flow over a stretching sheet has gained considerable attention due to its industrial applications. For instance, during the process of extrusion of a polymer sheet, from a die or in the drawing of plastic films, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. However, the quality of the final product of desired characteristics does not only depend on the rate of stretching but also on the rate of cooling. As such, a combination of both the flow caused by a stretching surface and heat transfer is of great importance in many manufacturing processes such as in the extrusion process, in glass blowing, hot rolling, the production of papers, the manufacture of plastic and rubber sheets, in crystal growing, in continuous cooling, in fibre spinning, etc.

The effects of such flows, along with electronic-magnetic fields, i.e. magnetohydrodynamic (MHD) flows, are relevant to many practical applications and play a major role industrially. The magnetic field has been widely used in the process of cooling continuous strips and filaments drawn through a quiescent fluid, the purification of molten metals and non-metallic inclusions, and has applications in different areas of research such as petroleum production and metallurgical processes. To the authors' knowledge, the MHD flow of non-Newtonian fluids was first studied by Sarpakaya (1961). Since then, there is an abundance of literature that discusses the MHD flows of non-Newtonian fluids over stretching sheet, and some of them can be found in Andersson (1992), Liao (2003), Eldabe and N.S. (2005), Sajid and Asghar (2007), Hayat and Qasim (201), Abel and Nandeppanavar (2008), Abel and Shinde (2012), Ashokkunar and Pravin (2013) and Aly and Vajravelu (2014).

Viscous dissipation changes the temperature distribution by playing the role of an energy source, which leads to affect heat transfer rates. The merit of the effect of viscous dissipation depends on whether the sheet is being cooled or heated (Abel and Ravikumara (2011)). As such, the viscous dissipation term has to be incorporated in the energy equation. Research in this field has been conducted by many investigators. Some of the recent ones are Abel and Ravikumara (2011) who investigated the effects of buoyancy, and viscous and Joule dissipation over a nonlinear vertical stretching porous sheet with partial slip in a Newtonian fluid. Jat and Chand (2013) studied the steady two-dimensional laminar flow of a viscous, incompressible electrically conducting fluid over an exponentially stretching sheet in the presence of a uniform transverse magnetic field with viscous dissipation and radiative heat flux. Ahmad and Nazar (2013) investigated the flow and heat transfer of a micropolar fluid past a nonlinearly stretching plate with viscous dissipation effect; and Pal and Vajravelu (2014) studied the mixed convection stagnation point flow of nanofluids over a stretching/shrinking surface in the presence of thermal radiation and viscous dissipation, to name a few.

In all these cases, a study of flow field and heat transfer can be significant because the quality of the final product depends to a large extent on the skin friction coefficient and the surface heat transfer rate (Bird and Hassager (1987)). Hence, motivated by the above works, the aim of this paper is to extend the above investigations, by which the velocity and heat transfer are calculated for the MHD flow of a Jeffrey fluid using a numerical approach.

2. Main Sections

Consider a steady, two-dimensional laminar boundary layer flow of an incompressible, electrically conducting Jeffrey fluid in the presence of a transverse magnetic field. The flow is generated by stretching the sheet away from the leading edge with linear velocity $u_w = ax$ where a is a positive constant. The plate is considered to have a temperature distribution in the quadratic form $T_w = T_\infty + A(x/L)^2$ at $y = 0$. The x-axis runs along the stretching sheet in the direction of motion, while the y-axis is taken normal to the sheet. A uniform magnetic field of strength B_o is applied in the positive direction of the y-axis. The magnetic Reynolds number is considered to be small so that the induced magnetic field is negligible. Under these assumptions, the governing boundary layer equations of the motion are

3. Equations

Mathematical equations should be numbered consecutively in the manuscript

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = \frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial y^2} + \lambda_2\left(u\frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right)\right] - \frac{\sigma B_o^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_o^2 u^2}{\rho c_p} \tag{3}$$

subject to the boundary conditions

$$\begin{aligned} &u = u_w, \quad v = 0 \quad T = T_w \text{at} \quad y = 0 \\ &u \to 0, \quad \frac{\partial u}{\partial y} \to 0, \quad T \to T_\infty \text{ as } y \to \infty \end{aligned} \tag{4}$$

where u and v are the velocity components in the x and y directions, respectively.ν is the kinematic viscosity, λ_1 is the ratio of the relaxation and retardation times, λ_2 is the relaxation time, k is the thermal conductivity , μ is the dynamic viscosity, c_p is the specific heat at constant pressure and T is the fluid temperature. ρ, σ and B_o are the fluid density, electric conductivity and magnetic field, respectively. The second term on the right side of Eq. (3) is the viscous dissipation term which is always positive and represents a source of heat due to friction between the fluid particles.

Eqs. (1)-(4) obey the following similarity transformation

$$\eta = \sqrt{\frac{a}{\nu}}\, y, \quad \psi = -\sqrt{a\nu}\, x f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where f is the dimensionless stream function, θ is the dimensionless temperature and ψ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Thus, we have

$$u = axf'(\eta), \quad v = -\sqrt{a\nu}\; f(\eta) \tag{6}$$

where prime denotes differentiation with respect to η . Invoking (5) and (6), Eq. (1) is automatically satisfied and Eqs. (2) and (3) are reduced to

$$f''' - (1+\lambda_1)\left[(f')^2 - ff''\right] + \beta\left[(f'')^2 - ff^{iv}\right] - (1+\lambda_1)\, Mf' = 0 \tag{7}$$

$$\theta'' + Pr\,(f\theta' - 2f'\theta) + PrEc\,(f'')^2 + MPrEc\,(f')^2 = 0 \tag{8}$$

and the transformed boundary conditions can be written as

$$\begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \;\; \text{at } \eta = 0 \\ f'(\eta) \to 0, \quad f''(\eta) \to 0, \quad \theta(\eta) \to 0 \;\; \text{as } \eta \to \infty \end{aligned} \tag{9}$$

where $\beta = a\lambda_2$ is the Deborah number,$Pr = \mu c_p/k$ is the Prandtl number, $Ec = a^2 l^2/Ac_p$ is the Eckert number and $M = \sigma B_o^2/\rho a$ is the MHD parameter.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2/2}, \quad Nu_x = \frac{xq_w}{k(T - T_\infty)}, \tag{10}$$

where τ_w is wall shear stress and q_w is the heat flux from the surface, which are given by

$$\tau_w = \mu \frac{\partial u}{\partial y}, \quad q_w = -k \frac{\partial T}{\partial y} \tag{11}$$

with μ and k being the dynamic viscosity and the thermal conductivity. Substituting (5) and (6) into (10), the scaled skin friction coefficient and the local Nusselt number are reduced to

$$\frac{1}{2} C_f Re_x^{1/2} = f''(0), \quad Nu_x Re_x^{-1/2} = -\theta'(0) \tag{12}$$

where $Re_x = u_w x/\nu$ is the local Reynolds number.

4. Results and Discussion

Eqs. (7) - (8), subject to the boundary conditions (9), have been solved numerically using a finite-difference method, namely the Keller-box method for some arbitrary values of the magnetic parameter M, the Jeffrey fluid parameter β , the Prandtl number Pr, and the Eckert number Ec with the ratio of the relaxation and retardation times λ_1 held fixed (=0).

Fig. 1 displays the variations of the local Nusselt number $-\theta'(0)$ with the Eckert number Ec when Pr = 0.7 and M= 0.2 for β = 0, 0.5 and 1.5, respectively. It is found that the local Nusselt number increases with the increase of Deborah number β and the local heat transfer is found to be higher for the low value of Ec. From our computation, we found that the values of the skin friction when M = 0.2 and β = 0, 0.5 and 1.5 for any values of Pr and Ec are -1.0955, -0.8945 and -0.6930, respectively. This indicates that the values of Ec and Pr have no significant impact on the skin friction coefficient.

The variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ with the Deborah number β when Pr = 0.7 and Ec = 0.3 are plotted in Figs. 2 and 3, respectively. Both of the figures suggest that as β increases, the skin friction and the heat transfer rate at the surface increases as well. It should be pointed out also, the effect of the magnetic parameter M is found to decrease both the skin friction coefficient and the local Nusselt number.

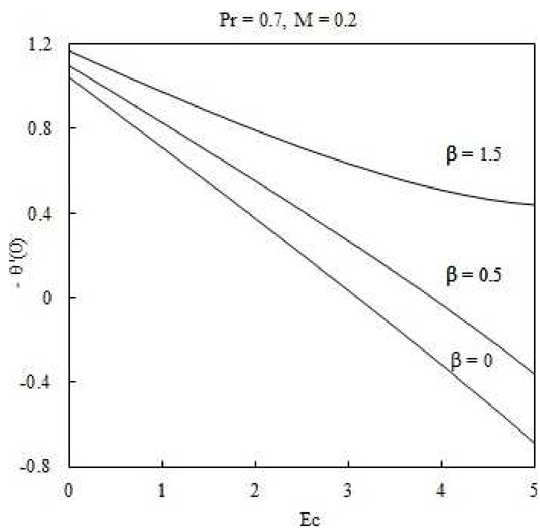

Figure 1: Variations of the local Nusselt number $-\theta'(0)$ with the Eckert number Ec when $Pr = 0.7$ and $M = 0.2$ for various values of Deborah number β

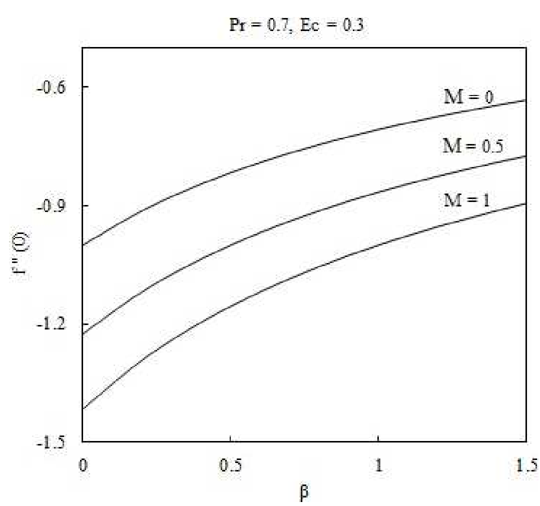

Figure 2: Variations of the skin friction coefficient $f''(0)$ with Deborah number β when $Pr = 0.7$ and $Ec = 0.3$ for various values of magnetic parameter M.

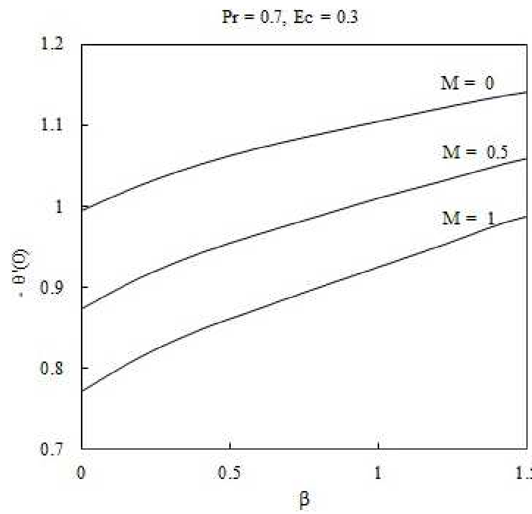

Figure 3: Variations of the local Nusselt number $-\theta'(0)$ with the Deborah number β when $Pr = 0.7$ and $Ec = 0.3$ for various values of magnetic parameter M.

Figs. 4 and 5 respectively show the influence of the Deborah number β on the velocity and temperature profiles when $Pr = 0.7$ and $Ec = 0.3$ for $M = 0$, 1. It is found that the velocity and the boundary layer thickness are increasing function of the Deborah number β. However, the temperature distribution shows an opposite trend as displayed in Fig. 5. The effect of the magnetic parameter M can also be seen from these two figures and it is found that the presence of M decreases the velocity and the boundary layer thickness but gives slight increment in the temperature inside the boundary layer. This is because the magnetic field presents as a damping effect, which retards the flow field by creating a drag force known as Lorentz force and hence opposes the fluid motion and reduces the velocity of the flow. In consequence of the retardation of the fluid flow and the resistance offered to the flow, it is expected to have an increment in the temperature as shown in Fig. 5.

Fig. 6 depicts the velocity distribution when $M = 0.5$ for the viscous fluid ($\beta = 0$) and the Jeffrey fluid ($\beta = 1$), respectively. The distribution of the velocity is observed to be identical for any values of Pr and Ec when the Deborah number β is fixed. This is expected as Pr and Ec only affect the thermal field, hence there is no significant impact on the flow field. As such, the thermal field is more pronounced compared to the velocity field as plotted in Fig. 7 and 8.

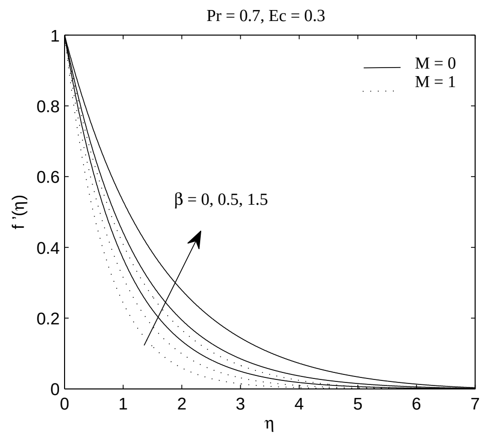

Figure 4: Velocity distribution for various values of Deborah number β when $Pr = 0.7$, $Ec = 0.3$ and $M = 0, 1$.

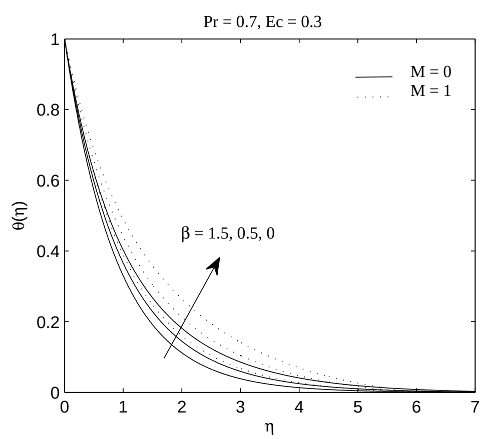

Figure 5: Temperature distribution for various values of Deborah number β when $Pr = 0.7$, $Ec = 0.3$ and, $M = 0, 1$.

The effect of Pr on the temperature distribution can be seen in Fig. 7, which demonstrates the variation of the temperature profiles for several values of Pr when $M = 0.5$ and $Ec = 1$ for the viscous and Jeffrey fluids, respectively. From the plot, it is evident that the temperature is at a fixed η and the thermal boundary layer thickness decreases rapidly as the boundary layer edge is reached faster with increasing value of Pr. Furthermore, fluid with a high Prandtl number decreases the thermal conductivities, which in turn retards the diffusion of the heat, and in consequence increases the temperature gradient at the surface. Also, it should be noted that the introduction of the material parameter/Deborah number β to the fluid is to reduce the temperature at the surface.

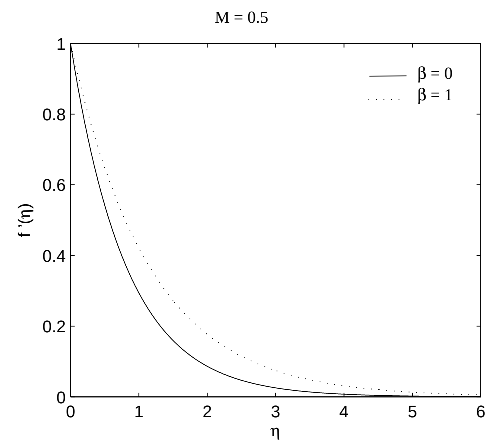

Figure 6: Velocity distribution when $M = 0.5$ and $\beta = 0,\ 1$.

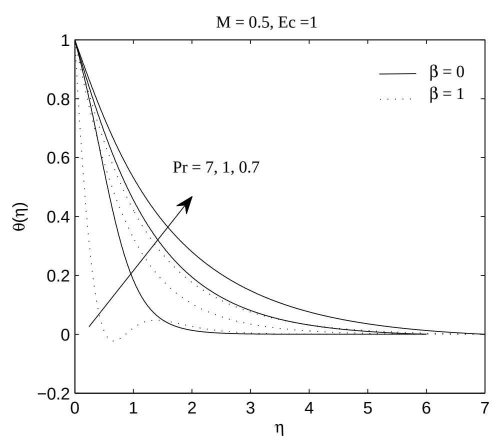

Figure 7: Temperature distribution for various values of Prandtl number Pr when $M = 0.5$, $Ec = 1$ and $\beta = 0,\ 1$.

The impact of the Eckert number on the temperature distributions of the viscous fluid (β=0) and the Jeffrey fluid (β=1) are shown in Fig. 8 with $Pr = 0.7$ and $M = 0.5$. It is observed that the temperature distribution increases with the increase of the Eckert number Ec. This is because the presence of viscous dissipation in the energy equation acts as an internal heat source that increase the thermal energy and thus heat the regime. For large values of Ec, it is evident that near the surface, the temperature slightly overshoots and adjourns marginally further into the boundary layer.

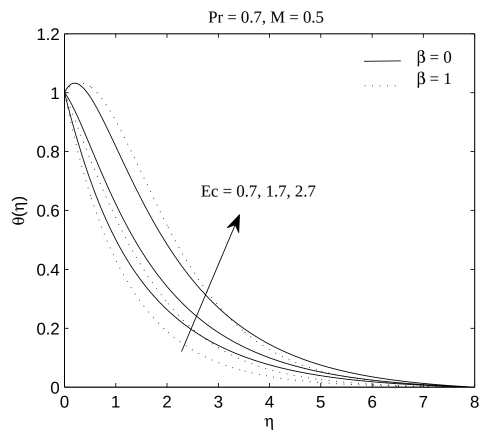

Figure 8: Temperature distribution for various values of Ec when $Pr = 0.7$, $M = 0.5$ and $\beta = 0$, 1.

For Figs. 4-8, it can be observed that the fluid velocity and temperature are high at the moving surface but then subside monotonously to zero as it achieves a certain distance far away from the plate surface, thus satisfying the boundary conditions (9).

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